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Multiple Attributes Decision Making by Extended Fuzzy Outranking Method with DSS Framework

Michihiro AMAGASA

Abstract

In this paper, the multiple attributes decision making has been proposed by the extended fuzzy outranking method based on the system modeling and discuss the framework for the decision support system of it, which makes it possible to perform evaluating and uniquely ranking the alternatives without losing the quality of data. In the outranking process of the proposed method, the uncertainty that adheres to decision making can be rationally handled with the concept of fuzziness. Therefore, this methodology provides a powerful systematic evaluation for dealing with the qualitative data in management decision making. The decision support system consists of five mathematical models, that is, the conversion model to triangular fuzzy number, the computation model of extended fuzzy outranking relation, the integrated fuzzy outranking relation model, the formulation model of fuzzy subordination matrix, the fuzzy outranking model by system modeling. Furthermore, in order to examine the effectiveness of the proposed method, a practical problem is studied as an empirical study, which is related to performance records for subjects of study.

Keywords: Multi-attribute decision making, Alternatives, Uncertainty, Fuzzy outranking relations, α-cut, System modeling, Decision support system

1. INTRODUCTION

In today when the values of the people have diversified, decision makers have to rationally evaluate and rank the alternatives taking into account of the various points of view, so called, the aspect of multi-attribute. In management decision making, the fuzzy outranking relations with vagueness are considered to be applicable to most objects (Zadeh 1965). In general, when the quantitative and qualitative data are mixed together, the qualitative data contain uncertainty in itself. Until now, the fuzzy outranking method (Roi 1991; Inoue and Amagasa et. al. 2008) and the multi-attribute decision-making (Siskos et al. 1986) have been proposed as the representative methods to evaluate and rank the alternatives. In these methods, at the early stages of ranking process, the qualitative data is changed into the quantitative one and dealt with in the same way as treated the other quantitative data. However, this will not lead to obtain a satisfied solution for the decision makers because they don't provide a convincing way for evaluating and ranking the alternatives with keeping the quality of data.

In this paper, we propose an extended fuzzy outranking method based on the system modeling (Nagata and Amagasa et. al. 2009) in decision making, which makes it possible to perform evaluating and uniquely ranking the alternatives without losing the quality of data. Further the framework of Decision Support System (DSS) is illustrated.

The fuzzy concept is involved in all of the qualitative data in regard to the attributes. The decision makers establish the fuzzy outranking relations between alternatives at each attribute, so that the elements of the relation are expressed in the form of the fuzzy membership functions of triangular type (Amagasa and Hirose 2012). Further, a synthesized fuzzy outranking relation is computed on the basis of the fuzzy outranking relations obtained in the previous step. In particular, if a difference between comparative alternatives is found to be little or alternatives are hard to compare each other, then the rule of α-cut (Amagasa and Hirose 2012) is applied without arranging unreasonable rankings, converting its subtle distinction into noticeable description. The synthesized fuzzy outranking relation is also formulated by the fuzzy membership function of triangular type.

This methodology provides a powerful systematic evaluation for dealing with the qualitative data in management decision making.

In the outranking process described above, the uncertainty that adheres to decision making can be rationally handled with the concept of fuzziness.

In order to examine the effectiveness of the proposed method, a practical problem is illustrated as an empirical study, which is related to performance records for subjects of study with the quantitative data and the qualitative data.

2. MATHEMATICAL PRELIMINARIES

We describe several properties with respect to the fuzzy number and its operation and mathematical tools as the mathematical preliminaries for the extended fuzzy outranking method in decision making.

2.1 Fuzzy number of triangular type

The fuzzy number of triangular type (Inoue and Amagasa 1998) is shown in (a_L, a_C, a_R), which satisfies $a_L \leq a_C \leq a_R$.

Here, a_L and a_C are, respectively, a left edge point, and a center point, and a_R a right edge point.

Then, the membership function of the fuzzy number of triangular type is defined by eq. (1) as follows.

$$\mu(x)=\begin{cases}0, & x\leq a_L\\ (x-a_L)/(a_C-a_L), & a_L<x<a_C\\ 1, & x=a_C\\ (a_R-x)/(a_R-a_C), & a_C<x<a_R\\ 0, & x\geq a_R\end{cases} \quad (1)$$

Furthermore, the fuzzy number of triangular type is classified as follows;

① $a_L\geq 0,\ a_R\geq 0$
② $a_L\leq 0,\ a_R\leq 0$
③ $a_L\leq 0,\ a_R\geq 0$

In this paper, we use the case of the fuzzy number of triangular type shown in (1). Here, we suppose two fuzzy number of triangular type by eq. (2) as follows.

$$A=(a_L,\ a_C,\ a_R),\ B=(b_L,\ b_C,\ b_R) \quad (2)$$

where $a_L\geq 0,\ a_R\geq 0,\ b_L\geq 0,\ b_R\geq 0$.

Then, the operations of A and B between the fuzzy numbers of triangular type are defined as follows.

① Addition:
$A+B=(a_L+b_L,\ a_C+b_C,\ a_R+b_R)$
② Subtraction:
$A-B=(a_L-b_R,\ a_C-b_C,\ a_R-b_L)$
③ Multiplication:
$k\cdot A=(k\cdot a_L,\ k\cdot a_C,\ k\cdot a_R)$, where k is a scalar.
$A\cdot B=(a_L\cdot b_L,\ a_C\cdot b_C,\ a_R\cdot b_R)$,
④ Division: $A/B=(a_L/b_R,\ a_C/b_C,\ a_R/b_L)$,

2.2 Transformations of quantitative data and qualitative data to the fuzzy number of triangular type

In this section, we formulate the fuzzy number of triangular type for qualitative data related to the performance records for subjects of study as shown in table 2.

The data S consists of the quantitative data and the qualitative data as follows;

$$S_i=\{A_i,\ B_i\},\ i=1, 2, ..., n$$

,where the symbol "n" shows the number of data.

The quantitative data with l attributes, A_i, $(i=1, 2, ..., l)$ and the qualitative data with m attributes, B_i, $(i=1, 2, ..., m)$ are, respectively, expressed as the fuzzy numbers of triangular type $(a_L,\ a_C,\ a_R)$ and $(b_L,\ b_C,\ b_R)$ with fuzziness.

The quantitative data A_i, $(i=1, 2, ..., l)$ is transformed to the fuzzy number of triangular type with "fuzzy parameter" "q" as follows;

$$A_i=(a_{i-q},\ a_i,\ a_{i+q}),\ i=1, 2, ..., n \quad (3)$$

,where the symbol "q" in eq. (3) shows the fuzziness given in advance by the decision makers.

Further, the qualitative data, B_i, $(i=1, 2, ..., m)$ is also expressed as the fuzzy number of triangular type with the "δ" recognition levels of decision making for the objects shown in table 1 (Edit and Refer to Miller 1956).

Table 1: Transformation the qualitative data to the fuzzy number of triangular type

Qualitative data	$B_i=(b_L,\ b_C,\ b_R),\ i=1, 2, ..., n$
Level 1	$(\delta-2/\delta-1, 1, 1)$
Level 2	$(\delta-3/\delta-1, \delta-2/\delta-1, 1)$
:	:
Level $\delta-1$	$(0, 1/\delta-1, 2/\delta-1)$
Level δ	$(0, 0, 1/\delta-1)$

2.3 Algorithm to make alternatives outrank with α-cut

In this section, the algorithm to make alternatives outrank is described as follows;

Step1: We find the intersection of two fuzzy numbers of triangular types which decides on outranking relation between two alternatives.

Step2: We find the α-cut value corresponding to a point of intersection obtained in step 1.

Step3: When the α-cut is greater than or equal to the value of intersection, we can discriminate the outranking relation between two alternatives.

For example, we try to compare A (2, 6, 10) and B (6, 10, 14) which are the fuzzy number of triangular type shown in figure 1.

At first we find a value of intersection and get $x=8$ then. Next, we find the α-cut value corresponding to a point of intersection and get $\alpha=0.5$.

In the case of $\alpha \geq 0.5$, B_α is superior to A_α. In other words, we can discriminate A and B by installing the α-cut.

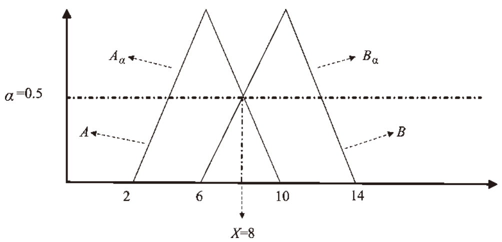


Figure 1: Illustrative example of α-cut ($\alpha=0.5$)

3. EXTENDED FUZZY OUTRANKING METHOD

When the quantitative and qualitative data are mixed together, the qualitative data contain uncertainty in itself. Until now, the fuzzy outranking method and the multi-attribute decision-making have been proposed as the representative methods to evaluate and rank the alternatives. In these methods, at the early stages of ranking process, the qualitative data is changed into the quantitative one and dealt with in the same way as treated the other quantitative data. However, this will not lead to obtain a satisfied solution for the decision makers because they don't provide a convincing way for evaluating and ranking the alternatives with keeping the quality of data.

In this paper, the extended fuzzy outranking method is proposed on the basis of the system modeling (Amagasa and Hirose 2012), which can pursue the solution reflected the real situation in.

There are various evaluation standards to evaluate the alternatives. In the case of a compound evaluation standard, the superiority and inferiority does not become clear, and contradiction happens for the superiority and inferiority relation, and it is not possible for evaluation easily. Such a slow superiority and inferiority relation is called fuzzy outranking relations, but there is much that the generosity becomes rather effective.
In this methodology, we handle uncertainty by transforming "qualitative data and quantitative data" having vagueness into

the fuzzy number of triangular type and derive the fuzzy outranking relation.

Then, the degree that can be convinced that the fuzzy number of triangular type, S (a_L, a_C, a_R) outranks S' (a'_L, a'_C, a'_R) is expressed in μ (S, S'). Then, this conviction degree satisfies the following inequality (4).

$$(0, 0, 0) \leq \mu(S, S') \leq (1, 1, 1). \quad (4)$$

(1) If we can convince that S completely outranks S', the following equations are satisfied.

When $a_L > a'_R$,

$\mu(S, S')=(1, 1, 1)$ and $\mu(S', S)=(0, 0, 0)$

(2) When $a'_R \geq a_L \geq a'_L$,

If α is greater than the value of the cross point of S and S', the following equation holds,

$\mu(S, S')=(1, 1, 1)$ and $\mu(S', S)=(0, 0, 0)$

Otherwise the relation of S and S' is indiscriminate, in the interval $[a_L, a'_R]$

$\mu(S, S')=(1, 1, 1)$ or $\mu(S', S)=(1, 1, 1)$

Then, the conviction degree of the extended fuzzy outranking relation is derived by eq. (5) on the basis of the operation rule for the fuzzy number of triangular type.

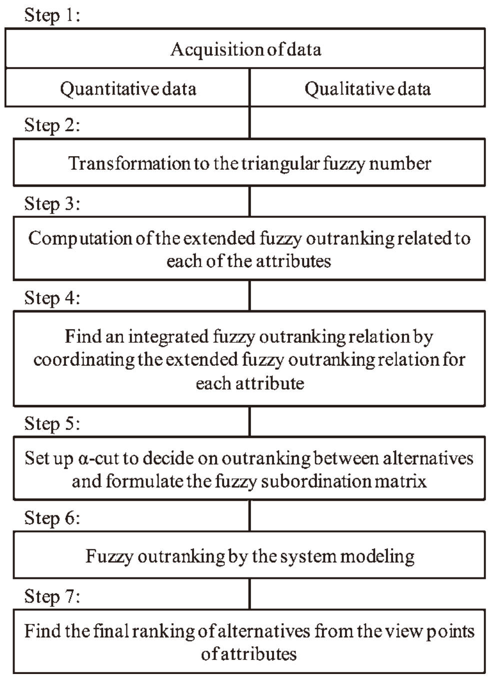
$$\mu(S, S')=S/S'=(a_L/a'_R, a_C/a'_C, a_R/a'_L) \quad (5)$$

$$a_C/a'_C \geq 1 \rightarrow a_C/a'_C=1$$

$$a_R/a'_L \geq 1 \rightarrow a_R/a'_L=1$$

4. DECISION MAKING METHOD WITH THE EXTENDED FUZZY OUTRANKING RELATION BASED ON THE STRUCTURAL MODELING

We propose a decision making method with the extended fuzzy outranking relation based on the system modeling as shown in figure 2 (Amagasa and Hirose 2012).



Remarks: "the triangular fuzzy number" shows "the fuzzy number of triangular type"

Figure 2: Decision making method with the extended fuzzy outranking relation based on the system modeling

In steps 1, 2 of figure 2, the data is acquired for the attributes and transform them to the fuzzy numbers of triangular type.

Further, the qualitative data represented with δ levels, that is, evaluation values for the alternatives, is expressed by the fuzzy number of triangular type, $B(a_L, a_C, a_R)$ shown in table 1.

In step 3, we compute the fuzzy outranking relations with respect to each of the $l+m$ attributes, that is, a conviction degree expressed by $\mu_k(S_i, S_j)$, $(k=1, 2, ..., n)$ $(i=1, 2, ..., l;\ j=1, 2, ..., m)$.

In step 4, the weight w_k, $(k=1, 2, ..., l+m)$ of the attributes is computed by the ratio method, where

$$\sum_{k=1}^{l+m} w_k = 1,\ 0 \leq w_k \leq 1,$$

and derive an integrated fuzzy outranking relation $\mu(S_i, S_j)$, $(i, j=1, 2, ..., n)$ by eq. (6), that is, an integrated conviction degree.

$$\mu(S_i, S_j) = \sum_{k=1}^{l+m} w_k \mu_k(S_i, S_j) \qquad (6)$$

In step 5, we set up α-cut to decide on outranking between alternatives and formulate the extended fuzzy outranking relation, that is, the fuzzy subordination matrix in system modeling.

In step 6, we identify the structural model based on the extended fuzzy outranking relation found in the previous step 5 by making use of the modified structural modeling method (Nagata and Amagasa et. al. 2009).

The structural model shows a result of outranking related to the alternatives.

In step 7, we find the final ranking of alternatives from the view points of attributes.

5. FRAMEWORK FOR THE DECISION SUPPORT SYSTEM

DSS is constructed from software system, database and model base. DSS software system is the body of DSS, and it is a set of software that manages the systems. It consists of dialogue generation management systems, database management systems and model base management systems.

In this paper, the decision support system for the multiple attributes decision making consists of five mathematical models as shown in figure 3, that is,

① the transformation model to triangular fuzzy number (Fuzzy transformation model),
② the computation model of extended fuzzy outranking relation (Extended fuzzy outranking model),
③ the integrated fuzzy outranking relation model,
④ the formulation model of fuzzy subordination matrix (Fuzzy subordination matrix model),
⑤ the fuzzy outranking model by system modeling (Fuzzy outranking model).

After this, we call the decision support system for the multiple attributes decision making by the extended fuzzy outranking "MADM-DSS".

The components of MADM-DSS consist of the inputs, the user knowledge and expertise, the outputs and the decision making as follows:

(1) Inputs: Input data, questions and mathematical models to analyze.
(2) User Knowledge and Expertise: Inputs

requiring manual analysis by users and/or decision makers.

(3) Outputs: Transformed data from which MADM-DSS "decisions" are generated.

(4) Decisions: Results generated by MADM-DSS based on user's criteria.

In general, the support given by DSS can be separated into three distinct, interrelated categories (Holsapple, C. W., Winston, A. B., 1996) as follows:

① Personal Support
② Group Support
③ Organizational Support

MADM-DSS here belongs to Personal and/or Group Support DSS.

MADM-DSS in this paper is represented in figure 3.

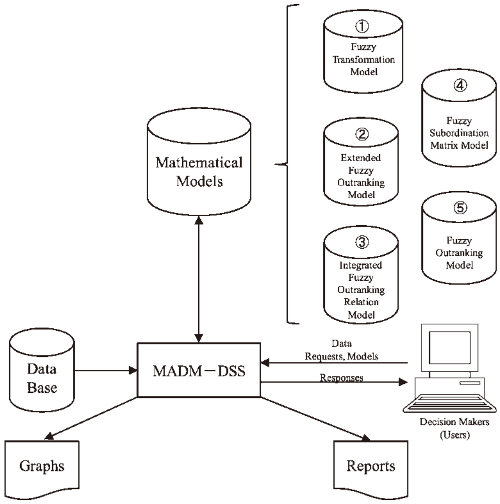


Figure 3: Structural model of MADM-DSS

The decision-makers and users enter data, questions and model into MADM-DSS and convey information. MADM-DSS transmits this information to mathematical models, that is,

① Fuzzy transformation model
② Extended fuzzy outranking model
③ Integrated fuzzy outranking relation model
④ Fuzzy subordination matrix model
⑤ Fuzzy outranking model

Then MADM-DSS answers the questions, processing the data and construct the models to adjust. After problem-solving, the solution is returned to users and/or decision makers from MADM-DSS, and resolution or response is returned to the decision-makers from MADM-DSS. At the same time, sentences, graphs and/or reporting can be created as output if it is required.

6. PRACTICAL APPLICATION

In this section, we illustrate a practical application of the extended fuzzy outranking method based on the system modeling as an empirical study, which is related to the performance records for subjects of study with the quantitative data and the qualitative data. Further we discuss about the difference between the results by the fuzzy outranking method and the extended fuzzy outranking method, which is based on system modeling proposed here, and examine the validity of the proposed method.

The problem solving process described in figure 2 is illustrated with the performance records for subjects of study with the quantitative data and the qualitative data given in advance as follows;

Steps 1, 2: We acquire the data of the performance records for subjects of study from three attributes as shown in table 2 and transform them to the fuzzy numbers of triangular types as shown in tables 3, 4 and 5.

Table 2: Performance records for subjects of study from three attributes

	g_1 A term-end examination	g_2 Report	g_3 Attitude of study
S_1	80	A	B
S_2	90	A	A
S_3	75	B	A
S_4	65	C	C
S_5	85	A	C

Table 3: A term-end examination

	A term-end examination	Fuzzy number of triangular type
S_1	80	(75, 80, 85)
S_2	90	(85, 90, 95)
S_3	75	(70, 75, 80)
S_4	65	(60, 65, 70)
S_5	85	(80, 85, 90)

Table 4: Report of study with 3 levels (A, B, C)

	Report	Fuzzy number of triangular type
S_1	A	(70, 85, 100)
S_2	A	(70, 85, 100)
S_3	B	(45, 60, 75)
S_4	C	(25, 40, 55)
S_5	A	(70, 85, 100)

Table 5: Attitude of study with 3 levels (A, B, C)

	Attitude of study	Fuzzy number of triangular type
S_1	B	(60, 70, 80)
S_2	A	(80, 90, 100)
S_3	A	(80, 90, 100)
S_4	C	(40, 50, 60)
S_5	C	(40, 50, 60)

Step 3: We compute the fuzzy outranking relation with respect to each of the three attributes shown in tables 3, 4 and 5 in step 2 and show in tables 6, 7 and 8, respectively.

Table 6: Extended fuzzy outranking relation from "A term-end examination" attribute (*R_1*)

	S_1	S_2	S_3	S_4	S_5
S_1	–	(0.79, 0.89, 1)	(0.94, 1, 1)	(1, 1, 1)	(0.83, 0.94, 1)
S_2	(1, 1, 1)	–	(1, 1, 1)	(1, 1, 1)	(0.94, 1, 1)
S_3	(0.82, 0.94, 1)	(0, 0, 0)	–	(1, 1, 1)	(0.78, 0.88, 1)
S_4	(0, 0, 0)	(0, 0, 0)	(0.75, 0.87, 1)	–	(0, 0, 0)
S_5	(0.94, 1, 1)	(0.84, 0.94, 1)	(1, 1, 1)	(1, 1, 1)	–

Table 7: Extended fuzzy outranking relation from the attribute "Report of study" (*R_2*)

	S_1	S_2	S_3	S_4	S_5
S_1	–	(0.70, 1, 1)	(0.93, 1, 1)	(1, 1, 1)	(0.70, 1, 1)
S_2	(0.70, 1, 1)	–	(0.93, 1, 1)	(1, 1, 1)	(0.70, 1, 1)
S_3	(0.45, 0.71, 1)	(0.45, 0.71, 1)	–	(0.82, 1, 1)	(0.45, 0.71, 1)
S_4	(0, 0, 0)	(0, 0, 0)	(0.33, 0.67, 1)	–	(0, 0, 0)
S_5	(0.70, 1, 1)	(0.70, 1, 1)	(0.93, 1, 1)	(1, 1, 1)	–

Table 8: Extended fuzzy outranking relation from the attribute "Attitude of study" (*R_3*)

	S_1	S_2	S_3	S_4	S_5
S_1	–	(0.60, 0.78, 1)	(0.60, 0.78, 1)	(1, 1, 1)	(1, 1, 1)
S_2	(1, 1, 1)	–	(0.8, 1, 1)	(1, 1, 1)	(1, 1, 1)
S_3	(1, 1, 1)	(0.8, 1, 1)	–	(1, 1, 1)	(1, 1, 1)
S_4	(0.5, 0.71, 1)	(0, 0, 0)	(0, 0, 0)	–	(0.67, 1, 1)
S_5	(0.5, 0.71, 1)	(0, 0, 0)	(0, 0, 0)	(0.67, 1, 1)	–

Step 4: We compute the weight w_k, ($k=1, 2, 3$) of the attributes by the ratio method. As the result of it, the weights for the attributes are obtained as follows;

$w_1=0.58 \quad w_2=0.25 \quad w_3=0.17$

Further, we derive an integrated fuzzy outranking relation R by $R=\sum_{k=1}^{3} w_k R_k$ and show it in table 9.

Table 9: Integrated extended fuzzy outranking relation from three attributes

$R=$

	S_1	S_2	S_3	S_4	S_5
S_1	–	(0.74, 0.90, 1)	(0.88, 0.96, 1)	(1, 1, 1)	(0.83, 0.97, 1)
S_2	(0.93, 1, 1)	–	(0.94, 1, 1)	(1, 1, 1)	(0.9, 1, 1)
S_3	(0.76, 0.90, 1)	(0.24, 0.35, 0.42)	–	(0.96, 1, 1)	(0.74, 0.86, 1)
S_4	(0.08, 0.12, 0.17)	(0, 0, 0)	(0.52, 0.68, 0.83)	–	(0.11, 0.17, 0.17)
S_5	(0.81, 0.95, 1)	(0.67, 0.80, 0.83)	(0.81, 0.83, 0.83)	(0.94, 1, 1)	–

Step 5: We set up α-cut to decide on outranking between alternatives and formulate the extended fuzzy outranking relation, that is, the fuzzy subordination matrix in system modeling as follows;

$$\alpha=\max_{i,j\in E} \alpha_{ij} \qquad (7)$$

,where E is a set of indexes showing all of the indiscriminate relations (S_i, S_j) between alternatives.

The α is recognized as a threshold installed to discriminate any relation between alternatives. Therefore the maximum value of α_{ij} in eq. (7) means a value that will be able to discriminate all of the indiscriminate relations between alternatives in the integrated fuzzy outranking relation.

The indiscriminate relations between alternatives in the integrated fuzzy outranking relation are picked up as follows;

(S_1, S_2), (S_1, S_3), (S_1, S_5), (S_3, S_5)

In order to discriminate the indiscriminate relations between alternatives, the α_{ij} are found as follows;

α_{12} for (S_1, S_2): 0.41
α_{13} for (S_1, S_3): 0.67
α_{15} for (S_1, S_5): 0.89
α_{35} for (S_3, S_5): 0.75

Then, α is computed by eq. (7).

$\alpha=\max\{\alpha_{12}, \alpha_{13}, \alpha_{15}, \alpha_{35}\}$
$=0.89$

Therefore, when we set up $\alpha=0.89$ in R, the conviction matrix is obtained as follows.

Conviction matrix

	S_1	S_2	S_3	S_4	S_5
S_1	–	0	1	1	1
S_2	1	–	1	1	1
S_3	0	0	–	1	1
S_4	0	0	0	–	0
S_5	0	0	0	1	–

The subordination matrix in the system modeling can be found by taking transposition of the conviction matrix described above as follows;

Subordination matrix

	S_1	S_2	S_3	S_4	S_5
S_1	–	1	0	0	0
S_2	0	–	0	0	0
S_3	1	1	–	0	0
S_4	1	1	1	–	1
S_5	1	1	1	0	–

We can identify the digraph, so called "a satisfied solution for the decision makers", for the given problem by making use of the modified structural modeling method.

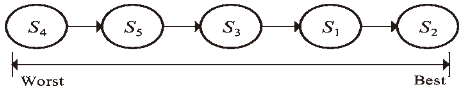


Figure 4: Digraph for the problem
(A satisfied solution: Ranking of performance records)

The result shown in figure 4 shows the ranking of performance records from the three attributes, which are "The term-end examination", "The report of study", and "The attitude of study". It is clear that the best evaluation is given for S_2 and the worst one S_4.

In this way, we can successively find a satisfied solution for decision makers and/or users by applying the proposed method to given problem. Further the result by the proposed method coincides with that one by the traditional fuzzy outranking method.

7. CONCLUSION

It is very important to determine the best alternatives (the satisfied solution) while taking into consideration the decision makers and/or the specialists' knowledge and opinion related to the given problem.

In this paper, as a method to solve the problem mentioned above, we proposed the extended fuzzy outranking method in decision making, which makes it possible to perform evaluating and uniquely ranking the alternatives without losing the quality of data. Furthermore, in order to examine the effectiveness of the proposed method, we studied a practical problem as an empirical study, which is related to the performance records for subjects of study.

As the results of it, the characteristics and merits of the proposed method were clear as follows;

(1) The decision makers can determine the best alternatives (the satisfied solution) while taking into consideration the decision makers and/or the specialists' knowledge and opinion related to the given problem.
(2) In comparison with the traditional method, the proposed method has a merit such that is able to find a satisfied solution uniquely and effectively.
(3) In case of a difference between comparative alternatives is found to be little and/or alternatives are hard to compare each other, then the rule of α-cut is applied without arranging unreasonable rankings, converting its subtle distinction into noticeable description. This methodology provides a powerful systematic evaluation for dealing with the qualitative data in management decision making.
(4) In the proposed outranking process, the uncertainty that adheres to decision making can be rationally handled with the concept of fuzziness.
(5) MADM-DSS framework suggests to be able to apply the proposed method to practical problems effectively and smoothly.

On the other hand, the following will be one of our future works. That is to say, we will have to apply the proposed method to a lot of the multiple attributes decision making problems as empirical studies in management decision making. It will be necessary to discuss about the results of it and to accumulate the know-how from the studies. These are being left as a subject in the future.

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